FURTHER STUDIES ON THE ORIGIN OF PLUTO

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ABSTRACT

An earlier hypothesis, that Pluto originated as a satellite of Neptune which was released during the stage of protoplanet evaporation, is confirmed by a study of the Jacobi constant of Pluto in the system sun-Neptune. The present value of the constant implies that the evaporation of the interior major protoplanets was not complete at the time of Pluto's release.

1. There are strong indications that Pluto originated as a satellite of Neptune which was released from its planetary bond after the evaporation of proto-Neptune had caused a reduction in Neptune's mass by the factor of roughly 40 (Kuiper 1956, p. 172). It remained to be demonstrated, however, that the Jacobi constant of Pluto in the system of sun and Neptune, which Rabe (personal communication) had computed to be 2.901, was compatible with this model.

The discussion may proceed on the basis of Rabe's (1954) treatment of the Trojan asteroids as escaped satellites from Jupiter, in which he introduced the concept of the osculating Jacobi integral and associated Jacobi index. A generalization of this analysis is needed in one respect. Rabe treated the solar mass as constant. In the present problem it is advisable to consider the possibility of a decreasing solar mass. The reason is not that the sun itself is supposed to vary; as was emphasized by Kuiper (1956, p. 64), solar-mass losses during geologic time have probably been negligible, although a dynamical verification of this extrapolation would, of course, be most desirable. The principal reason for expecting an effective decrease in the solar mass is that Jupiter and Saturn are sufficiently close to the sun compared to Neptune that, to a first approximation, they will affect the orbits of Neptune and Pluto as if they were part of the sun itself. The total mass decrease of the inner part of the solar system is estimated to have been about 0.05–0.06 solar masses (Kuiper 1956); a total mass decrease of the central body by 5–6 per cent is therefore to be expected.

The discussion of the Jacobi integral for Pluto involves, therefore, mass decreases by both the sun and proto-Neptune. Rabe's equation (10) may now be generalized to read

$$\frac{d\,(V^2)}{dt} = 2\,\frac{d\Omega_0}{dt} + \Delta\mu_s\,\frac{d}{dt}\left(\frac{2}{r} + r^2\right) + \Delta\mu_N\,\frac{d}{dt}\left(\frac{2}{\rho} + \rho^2\right), \tag{1}$$

in which $\Delta\mu_s$ and $\Delta\mu_N$ are the finite mass changes of the sun and Neptune in terms of the initial solar mass, valid at time t; r is the distance of Pluto from the sun; ρ the distance from Neptune; V the velocity of Pluto in the co-ordinate system rotating with Neptune and the sun; and Ω_0 given by

$$2\Omega_0 = \frac{2}{r} + r^2 + \mu_0\left(\frac{2}{\rho} + \rho^2\right),$$

in which μ_0 is the initial mass of proto-Neptune. The osculating Jacobi integral (Rabe's eq. [12]) now becomes

$$V^2 = 2\Omega - C = 2\Omega_0 + \Delta\mu_s\left(\frac{2}{r} + r^2\right) + \Delta\mu_N\left(\frac{2}{\rho} + \rho^2\right) - C, \tag{2}$$

so that

$$\frac{d(V^2)}{dt} = 2\frac{d\Omega_0}{dt} + \Delta\mu_s \frac{d}{dt}\left(\frac{2}{r} + r^2\right) + \Delta\mu_N \frac{d}{dt}\left(\frac{2}{\rho} + \rho^2\right) + \left(\frac{2}{r} + r\right)\frac{d\mu_s}{dt} + \left(\frac{2}{\rho} + \rho^2\right)\frac{d\mu_N}{dt} - \frac{dC}{dt}, \quad (3)$$

and C is found from

$$\frac{dC}{dt} = \left(\frac{2}{r} + r^2\right)\frac{d\mu_s}{dt} + \left(\frac{2}{\rho} + \rho^2\right)\frac{d\mu_N}{dt}. \quad (4)$$

We consider two limiting cases:

a) $d\mu_N/dt = 0$ after Pluto was set free. This may not be far from the truth, since Neptune's total mass loss may have been by a factor of 100–150 (Kuiper 1956), while the mass loss suffered before Pluto was released was already at least 40, because the factor 40 applies only after Pluto had condensed into a satellite and acquired its present period of rotation.

b) $d\mu_s/dt = 0$. This assumes that Jupiter and Saturn had already lost nearly all their mass at the time when Pluto was released.

The principal object of this paper is to determine which alternative best explains the present Pluto orbit.

2. Assumption *a* leads to

$$\frac{dC}{d\mu_s} = \frac{2}{r} + r^2 \cong 3. \quad (5)$$

Now the value of C just after Pluto was released must have been slightly in excess of 3. Since proto-Neptune's mass at that time was probably about 0.00015 solar masses or somewhat less, the radius of action of Neptune was about 0.028 units. If Pluto were released at that distance from Neptune and at unit distance from the sun, with negligible velocity in the rotating frame, the value of C was 3.011. Hence we may adopt $\Delta C = -0.110$, from which follows

$$\Delta\mu_s = -0.037\,.$$

This is compatible with the assumptions that *Neptune lost little mass after Pluto was set free, while proto-Jupiter and proto-Saturn still possessed most of their original masses.*

3. Assumption *b* leads to

$$\frac{dC}{d\mu_N} = \frac{2}{\rho} + \rho^2. \quad (6)$$

Now Neptune's mass loss due to evaporation by solar radiation will be very nearly uniform along its nearly circular orbit. Therefore, we shall be able to derive ΔC as a function $\Delta\mu_N$ from the *time average* of $(2/\rho + \rho^2)$. Such an average can be derived accurately only from numerical integrations, but a good estimate may be made from a simple model according to which Pluto has a circular orbit around the sun, in the same plane as Neptune, with a semimajor axis $a_P = a_N + \Delta$, in which a_N is unit distance and Δ may be, say, 10^{-2}–10^{-1}. It may be expected that Δ will change during the mass decrease of Neptune; the present value would be 0.3.

For small values of Δ, the quantity (6) will run from the value of $2/\Delta$ at conjunction, to 3 at quadrature, to 5 at opposition. Its time average value, A, may therefore be expected to be of the order of 5. The average will now be determined more accurately. If α is the heliocentric angle between Neptune and Pluto,

$$\rho^2 = 1 + (1+\Delta)^2 - 2(1+\Delta)\cos\alpha = a - b\cos\alpha\,, \tag{7}$$

in which $a = 1 + (1+\Delta)^2 \cong 2(1+\Delta) \cong 2$; and $b = 2(1+\Delta) \cong 2$. The ratio $c = b/a = 1 - \epsilon$, where $\epsilon = \frac{1}{2}\Delta^2 +$ higher-order terms. Now

$$A = \frac{2}{\pi}\int_0^{\pi} \frac{d\alpha}{(a - b\cos\alpha)^{1/2}} + \frac{1}{\pi}\int_0^{\pi}(a - b\cos\alpha)\,d\alpha = S_1 + a\,.$$

If $\beta = \alpha/2$ and $\delta^2 = \epsilon/2c$, and hence $\delta \cong \Delta/2$,

$$S_1 = \frac{2}{\pi}\sqrt{\frac{2}{ac}}\int_0^{\pi/2}\frac{d\beta}{(\delta^2 + \sin^2\beta)^{1/2}} \cong \frac{2}{\pi}\left(1 - \frac{\Delta}{2}\right)\int_0^{\pi/2}\frac{dx}{(\delta^2 + x^2)^{1/2}} \cong \frac{2}{\pi}\left(1 - \frac{\Delta}{2}\right)\ln\frac{2\pi}{\Delta}\,;$$

or

$$A \cong 1 + (1+\Delta)^2 + \frac{2}{\pi}\left(1 - \frac{\Delta}{2}\right)\ln\frac{2\pi}{\Delta}\,. \tag{8}$$

For $\Delta = 0.01$, $A = 6.10$; for $\Delta = 0.10$, $A = 4.71$; for $\Delta = 0.15$, $A = 4.52$. Because of the logarithmic dependence on Δ, the precise value of Δ is not important, as long as Δ is small. Adopting $A = 5$, we find from $\Delta C = -0.1$ that $\Delta\mu_N = 0.02$. Since the original mass of proto-Neptune was only about 0.006 (Kuiper 1956) and the mass at the time of Pluto's release at most 1/40 of that, or roughly 10^{-4}, it follows that assumption *b* is unsuited. Further, since effect *b* is able to account for only a very small decrease in *C*, assumption *a* must be closely correct.

4. The conclusion just reached has a bearing on the general theory of planet formation, since it connects the early evolution in two widely separated parts of the solar system. An earlier example of the same was Rabe's (1956) discovery that the gaps in the asteroid ring are to be attributed to massive proto-Jupiter, not the modern planet. The time scales of protoplanet evaporation are difficult to compute accurately, and the new result gives semiempirical evidence on the relative time schedules of proto-Neptune and the two main interior planets (the mass losses of the terrestrial planets and the asteroid ring are, of course, included, but they are only some 10 per cent of the total). It is very satisfactory that the general nature of Pluto's orbit is found to be a direct consequence of processes in the solar system already postulated on the basis of other evidence. The result appears to establish Pluto's origin as a former satellite of Neptune; but it does not yet give a complete explanation of the present *a*, *e*, and *i* values of its orbit, only the combination of these elements occurring in the Jacobi constant, i.e., approximately $a^{-1} + 2[a(1-e^2)]^{1/2}\cos i$.

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